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$\mathscr{Z}$-symmetries of (ε)-para-Sasakian 3-manifolds

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Abstract

The object of this paper is study (ε)-para-Sasakian 3-manifolds satisfying certain conditions on the $\mathscr{Z}$ tensor. We characterize: $\mathscr{Z}$-symmetric, $\mathscr{Z}$-semisymmetric, $\mathscr{Z}$-pseudosymmetric, and projectively $\mathscr{Z}$-semisymmetric conditions on an (ε)-para-Sasakian 3-manifold.

Keywords

(ε)-para-Sasakian 3-manifold, $\mathscr{Z}$ tensor, $\mathscr{Z}$-semisymmetric, $\mathscr{Z}$-pseudosymmetric, Ricci-symmetric, Ricci-semisymmetric, Einstein manifold.

AMS Subject Classification

53C15, 53C25.

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Contents

1. Introduction

In 1969, Takahashi [1] initiated the study of almost contact manifolds associated with an indefinite metrics. These indefinite almost contact metric manifolds are also called as (ε)-almost contact metric manifolds. The study of indefinite metric manifolds is of interest from the standpoint of physics and relativity. Indefinite metric manifolds have been studied by several authors. In 1993, Bejancu and Duggal [2] introduced the concept of (ε)-Sasakian manifolds. Some interesting properties of these manifolds was studied in the papers [3], [4], [5], [6] and the references therein. In 2009, De and Sarkar [7] introduced the concept of (ε)-Kenmotsu manifolds and showed that the existence of new structure on an indefinite metrics influences the curvatures.

Tripathi and his co-authors [8] initiated the study of (ε)-almost para-contact metric manifolds, which is not necessarily Lorentzian. In particular, they studied (ε)-para-Sasakian manifolds, with the structure vector field ξ is spacelike or timelike according as $\varepsilon = 1$ or $\varepsilon = -1$. An (ε)-almost contact metric manifold is always odd dimensional but an (ε)-almost para-contact metric manifold could be even dimensional as well. Later, Perktas and his co-authors [9] studied (ε)-para-Sasakian manifolds in dimension 3.

In 2012, Mantica and Molinari [10] defined a generalized (0, 2) symmetric $\mathscr{Z}$ tensor given by

$$\mathscr{Z}(X,Y) = S(X,Y) + \psi g(X,Y), \tag{1.1}$$

where ψ is an arbitrary scalar function. Properties of $\mathscr{Z}$ tensor were pointed out in the papers [11] and [12]. This tensor is a general notion of the Einstein gravitational tensor in General Relativity. Recently, Mallick and De [13] studied various properties of $\mathscr{Z}$ tensor on N(k)-quasi-Einstein manifolds.

The present paper is organized as follows: After preliminaries, in Section 3, we study $\mathscr{Z}$-semisymmetric, $\mathscr{Z}$-pseudosymmetric and projectively $\mathscr{Z}$-semisymmetric (ε)-para-Sasakian 3-manifolds. Here, we prove that, for an (ε)-para-Sasakian 3-manifold, the conditions of being $\mathscr{Z}$-semisymmetric; Ricci-symmetric; Ricci-semi symmetric; or Einstein manifold are all equivalent. Also show that, if an (ε)-para-Sasakian 3-manifold M is $\mathscr{Z}$-pseudosymmetric then it is either Ricci-semisymmetric or pseudosymmetric function $L_{\mathscr{Z}} = -\varepsilon$ holds on M. Further, we prove that a projectively $\mathscr{Z}$-semisymmetric (ε)-para-Sasakian 3-manifold is an Einstein manifold.

2. Preliminaries

A manifold M is to admit an almost para-contact structure if it admit a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η satisfying

$$\phi^2 = I - \eta \otimes \xi,\ \eta(\xi) = 1,\ \phi\xi = 0,\ \eta \cdot \phi = 0. \quad (2.1)$$

Let g be a semi-Riemannian metric with index$(g) = v$ such that

$$g(\phi X, \phi Y) = g(X,Y) - \varepsilon\eta(X)\eta(Y).\ X,Y \in TM, \quad (2.2)$$

where, $\varepsilon = \pm 1$. Then M is called an (ε)-almost para-contact metric manifold equipped with an (ε)-*almost para-contact metric structure* $(\phi, \xi, \eta, g, \varepsilon)$. In particular, if index of g is equal to one, then an (ε)-almost para-contact metric manifold is said to be a *Lorentzian almost para-contact manifold*. In particular, if the metric g is positive definite, then (ε)-almost para-contact metric manifold is the usual *almost para-contact metric manifold* [14].
The equation (2.2) implies that

$$g(X, \phi Y) = g(\phi X, Y) \ \textit{and}\ g(X,\xi) = \varepsilon\eta(X). \quad (2.3)$$

From (2.1) and (2.3) it follows that

$$g(\xi,\xi) = \varepsilon. \quad (2.4)$$

An (ε)-almost para-contact metric structure is called an (ε)-para-Sasakian structure if

$$(\nabla_X \phi)Y = -g(\phi X, \phi Y)\xi - \varepsilon\eta(Y)\phi^2 X.\ X,Y \in TM, \quad (2.5)$$

where ∇ is the Levi-Civita connection with respect to g. A manifold endowed with (ε)-para-Sasakian structure is called an (ε)-*para-Sasakian manifold* [8].

For $\varepsilon = 1$ and g Riemannian, M is the usual para-Sasakian manifold [15], [16]. For $\varepsilon = -1$, g Lorentzian and ξ replace by $-\xi$, M becomes a Lorentzian para-Sasakian manifold [17].

For an (ε)-para-Sasakian manifold, it is easy to prove that

$$\begin{aligned} R(X,Y)\xi &= \eta(X)Y - \eta(Y)X, & (2.6)\\ R(\xi,X)Y &= \eta(Y)X - \varepsilon g(X,Y)\xi, & (2.7)\\ R(\xi,X)\xi &= X - \eta(X)\xi, & (2.8)\\ S(X,\xi) &= -(n-1)\eta(X), & (2.9)\\ \nabla_X\xi &= \varepsilon\phi X. & (2.10)\end{aligned}$$

For detail study of (ε)-para-Sasakian manifold, see [8].

It is known that in a 3-dimensional (ε)-para-Sasakian manifold (or, an (ε)-para-Sasakian 3-manifold), the Riemannian curvature tensor and the Ricci tensor has the following form [9]:

$$\begin{aligned} R(X,Y)Z &= \left(\frac{r}{2}+2\varepsilon\right)\{g(Y,Z)X - g(X,Z)Y\} \\ &- \left(\frac{r}{2}+3\varepsilon\right)\{g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ &+ \varepsilon\eta(Y)\eta(Z)X - \varepsilon\eta(X)\eta(Z)Y\}, \quad (2.11)\\ S(X,Y) &= \left(\frac{r}{2}+\varepsilon\right)g(X,Y) - \varepsilon\left(\frac{r}{2}+3\varepsilon\right)\eta(X)\eta(Y). \\ & \quad (2.12)\end{aligned}$$

The projective curvature tensor $\mathscr{P}$ in a (ε)-para-Sasakian 3-manifold M is defined by

$$\mathscr{P}(X,Y)U = R(X,Y)U - \frac{1}{2}[S(Y,U)X - S(X,U)Y]. \quad (2.13)$$

In an (ε)-para-Sasakian 3-manifold M, the $\mathscr{Z}$ tensor takes the form

$$\mathscr{Z}(X,Y) = \left(\frac{r}{2}+\varepsilon+\psi\right)g(X,Y) - \varepsilon\left(\frac{r}{2}+3\varepsilon\right)\eta(X)\eta(Y). \quad (2.14)$$

and scalar $\mathscr{Z}$ takes the form

$$\mathscr{Z} = \left(\frac{r}{2}+\varepsilon+\psi\right)3 - \left(\frac{r}{2}+3\varepsilon\right) = r + 3\psi.$$

Also,

$$\mathscr{Z}(X,\xi) = (\varepsilon\psi - 2)\eta(X). \quad (2.15)$$

3. $\mathscr{Z}$-symmetries of (ε)-para-Sasakian 3-manifolds

In this section, we characterize, $\mathscr{Z}$-symmetric; $\mathscr{Z}$-semisymmetric; $\mathscr{Z}$-pseudosymmetric; and projectively $\mathscr{Z}$-semisymmetric conditions on an (ε)-para-Sasakian 3-manifolds. We begin with the following:

Definition 3.1. *A semi-Riemannian manifold M is called locally symmetric if its curvature tensor R is para-llel, that is, $\nabla R = 0$, where ∇ denotes the Levi-Civita connection. As a proper generalization of locally symmetric manifolds, the notion of semisymmetric manifolds was defined by*

$$R(X,Y)\cdot R = 0,$$

for any $X,Y \in TM$. A complete intrinsic classification of these spaces was given by Szabo [18].

Definition 3.2. *A semi-Riemannian manifold M is said to be $\mathscr{Z}$-symmetric if $\nabla\mathscr{Z} = 0$, and it is called $\mathscr{Z}$-semisymmetric if*

$$R(X,Y)\cdot\mathscr{Z} = 0, \quad (3.1)$$

for any $X,Y \in TM$, where $R(X,Y)$ acts as a derivation on $\mathscr{Z}$.

Let M be a $\mathscr{Z}$-semisymmetric (ε)-para-Sasakian 3-manifold. Then from (3.1), we have

$$\mathscr{Z}(R(X,Y)U,V) + \mathscr{Z}(U,R(X,Y)V) = 0.$$

In particular,

$$\mathscr{Z}(R(\xi,Y)U,V) + \mathscr{Z}(U,R(\xi,Y)V) = 0. \quad (3.2)$$

From (2.7), we obtain

$$\mathscr{Z}(R(\xi,Y)U,V) = -\varepsilon g(Y,V)\mathscr{Z}(U,\xi) + \mathscr{Z}(U,Y)\eta(V)$$

(3.3)

and

$$\mathscr{Z}(U,R(\xi,Y)V) = -\varepsilon g(Y,U)\mathscr{Z}(V,\xi) + \mathscr{Z}(V,Y)\eta(U). \tag{3.4}$$

The equations (3.2), (3.3) and (3.4) together give

$$\begin{aligned} &-\varepsilon\{g(Y,U)\mathscr{Z}(V,\xi) + g(Y,V)\mathscr{Z}(U,\xi)\} \\ &+\mathscr{Z}(V,Y)\eta(U) + \mathscr{Z}(U,Y)\eta(V) = 0. \end{aligned} \tag{3.5}$$

Setting $V = \xi$ in (3.5) and using (2.15), we have

$$\mathscr{Z}(U,Y) = (\psi - 2\varepsilon)g(Y,U). \tag{3.6}$$

By making use of (2.14) in (3.6), we obtain

$$\left(\frac{r}{2} + 3\varepsilon\right)[g(Y,U) - \varepsilon\eta(Y)\eta(U)] = 0. \tag{3.7}$$

Since $g(Y,U) - \varepsilon\eta(Y)\eta(U) = g(\phi Y, \phi U) \neq 0$, in general, therefore we obtain from (3.7) that $\frac{r}{2} + 3\varepsilon = 0$, that is,

$$r = -6\varepsilon. \tag{3.8}$$

Next, using (3.8) in (2.12) we get

$$S(X,Y) = -2\varepsilon g(X,Y). \tag{3.9}$$

That is, M is an Einstein manifold.
Conversely, suppose that the manifold M be an Einstein. Then, from (1.1) and (3.9) we have (3.6). Next, consider

$$\begin{aligned} &R(X,Y)\cdot\mathscr{Z}(U,V) \\ =\ &\mathscr{Z}(R(X,Y)U,V) + \mathscr{Z}(U,R(X,Y)V). \end{aligned} \tag{3.10}$$

By using (3.6) in (3.10) we obtain

$$\begin{aligned} R(X,Y)\cdot\mathscr{Z}(U,V) &= \\ (\psi - 2\varepsilon)\{g(R(X,Y)U,V) & \\ +g(U,R(X,Y)V)\}. & \end{aligned} \tag{3.11}$$

It is known that, in an (ε)-para-Sasakian manifold, the following relation holds:

$$(R(X,Y)U,V) = -g(R(X,Y)V,U). \tag{3.12}$$

From (3.11) and (3.12), it follows that

$$R(X,Y)\cdot\mathscr{Z}(U,V) = 0.$$

That is, M is $\mathscr{Z}$-semisymmetric. Hence, we are able to state the following result:

Theorem 3.3. *An (ε)-para-Sasakian manifold is $\mathscr{Z}$-semisymmetric if and only if it is an Einstein manifold.*

Further, the manifold M is an Einstein, implies trivially that M is Ricci-symmetric.

Conversely, if M is Ricci-symmetric, that is, $\nabla S = 0$. In particular,

$$(\nabla_X S)(Y,\xi) = \varepsilon S(\phi X,Y) + 2g(\phi X,Y) = 0.$$

Replacing X by ϕX in the above equation, shows that the manifold is an Einstein manifold. Therefore, by taking into account of theorem 3.3, we state the following:

Theorem 3.4. *An (ε)-para-Sasakian 3-manifold is $\mathscr{Z}$-semisymmetric if and only if it is Ricci-symmetric.*

Moreover, suppose that M be Ricci-semisymmetric, that is,

$$(R(X,Y)\cdot S)(U,V) = 0.$$

In particular,

$$(R(\xi,Y)\cdot S)(U,\xi) = 0,$$

this implies that

$$S(R(\xi,Y)U,\xi) + S(U,R(\xi,Y)\xi) = 0,$$

which in view of (2.7) and (2.12) gives (3.9).

Conversely, if M is an Einstein manifold, then obviously, it is Ricci-semisymmetric. Thus, the manifold M is Ricci-semisymmetric if and only if it is an Einstein manifold. Hence, by taking into account of theorem 3.3, we have the following:

Theorem 3.5. *An (ε)-para-Sasakian 3-manifold is $\mathscr{Z}$-semisymmetric if and only if it is Ricci-semisymmetric.*

Corollary 3.6. *In an (ε)-para-Sasakian 3-manifold, the following statements are equivalent:*

1. *M is an Einstein manifold.*
2. *M is Ricci-symmetric.*
3. *M is Ricci-semisymmetric.*
4. *M is $\mathscr{Z}$-semisymmetric.*

It is clear that, $\nabla\mathscr{Z} = 0 \Rightarrow R\cdot\mathscr{Z} = 0 \Rightarrow \nabla S = 0$. Therefore, from corollary 3.6, we get:

Corollary 3.7. *Every $\mathscr{Z}$-symmetric (ε)-para-Sasakian 3-manifold is Ricci-symmetric.*

For a $(0,k)$-tensor field T on M, $k \geq 1$, and a symmetric $(0,2)$-tensor field A on M, we define the $(0,k+2)$-tensor fields $R\cdot T$ and $Q(A,T)$ by

$$\begin{aligned} &(R\cdot T)(X_1,...,X_k;X,Y) \\ =\ &-T(R(X,Y)X_1,X_2,...,X_k) - ... - T(X_1,...,X_{k-1},R(X,Y)X_k) \end{aligned}$$

and

$$\begin{aligned} &Q(A,T)(X_1,...,X_k;X,Y) \\ =\ &-T((X\wedge_A Y)X_1,X_2,...,X_k) - ... - T(X_1,...,X_{k-1},(X\wedge_A Y)X_k) \end{aligned}$$

respectively, where $X\wedge_A Y$ is the endomorphism given by

$$(X\wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{3.13}$$

Definition 3.8. *A semi-Riemannian M is said to be pseudosymmetric (in the sense of R. Deszcz [19]) if*

$$R\cdot R = L_R Q(g,R)$$

holds on $U_R = \{x\in M : R\neq 0\frac{r}{n(n-1)}G \ \text{at} \ x\}$, where G is the $(0,4)$-tensor defined by $G(X_1,X_2,X_3,X_4)=g((X_1\wedge X_2)X_3,X_4)$ and L_R is some function on U_R.

Definition 3.9. *A semi-Riemannian manifold M is said to be $\mathscr{Z}$-pseudosymmetric if*

$$(R(X,Y)\cdot\mathscr{Z})(U,V) = L_{\mathscr{Z}}Q(g,\mathscr{Z})(U,V;X,Y) \quad (3.14)$$

holds on the set $U_{\mathscr{Z}} = \{x\in M : \mathscr{Z}\neq 0 \ \text{at} \ x\}$, and $L_{\mathscr{Z}}$ is some function on $U_{\mathscr{Z}}$.

Let M be a $\mathscr{Z}$-pseudosymmetric (ε)-para-Sasakian 3-manifold. Then from (3.13) and (3.14), we have

$$(R(\xi,Y)\cdot\mathscr{Z})(U,\xi) = L_{\mathscr{Z}}\left[((\xi\wedge Y)\cdot\mathscr{Z})(U,\xi)\right]. \quad (3.15)$$

In an (ε)-para-Sasakian 3-manifold, from (2.7) and (3.13) we get

$$R(\xi,X)Y = (-\varepsilon)(\xi\wedge X)Y. \quad (3.16)$$

In view of (3.15) in (3.16), it is easy to see that

$$L_{\mathscr{Z}} = -\varepsilon.$$

Hence, by taking into account of previous calculations and discussions, we conclude the following:

Definition 3.10. *Let M be an (ε)-para-Sasakian 3-manifold. If M is $\mathscr{Z}$-pseusosymmetric, then either M is an Einstein manifold or $L_{\mathscr{Z}} = -\varepsilon$ holds on M.*

If $L_{\mathscr{Z}}\neq -\varepsilon$, then immediately, we obtain the following:

Corollary 3.11. *Every $\mathscr{Z}$-pseudosymmetric (ε)-para-Sasakian 3-manifold with $L_{\mathscr{Z}}\neq-\varepsilon$ is an Einstein manifold.*

But $L_{\mathscr{Z}}$ need not be zero, in general and hence there exists $\mathscr{Z}$-pseudosymmetric manifolds which are not $\mathscr{Z}$-semisymmetric. Thus the class of $\mathscr{Z}$-pseudosymmetric manifolds is a natural extension of the class of $\mathscr{Z}$-semisymmetric manifolds. Thus, if $L_{\mathscr{Z}}\neq 0$ then it is easy to see that $R\cdot\mathscr{Z} = (-\varepsilon)Q(g,\mathscr{Z})$, which implies that the pseudosymmetric function $L_{\mathscr{Z}} = -\varepsilon$. Therefore, we able to state the following result:

Corollary 3.12. *Every (ε)-para-Sasakian 3-manifold is $\mathscr{Z}$-pseudosymmetric of the form $R\cdot\mathscr{Z} = (-\varepsilon)Q(g,\mathscr{Z})$.*

Definition 3.13. *A semi-Riemannian manifold M is said to be projectively $\mathscr{Z}$-semisymmetric if*

$$\mathscr{P}(X,Y)\cdot\mathscr{Z} = 0, \quad (3.17)$$

for any $X,Y\in TM$, where $\mathscr{P}$ is the projective curvature tensor.

Let M be a projectively $\mathscr{Z}$-semisymmetric (ε)-para-Sasakian 3-manifold. Then from (3.17), we have

$$\mathscr{Z}(\mathscr{P}(X,Y)U,V) + \mathscr{Z}(U,\mathscr{P}(X,Y)V) = 0.$$

This implies

$$\mathscr{Z}(\mathscr{P}(\xi,Y)U,V) + \mathscr{Z}(U,\mathscr{P}(\xi,Y)V) = 0. \quad (3.18)$$

In an (ε)-para-Sasakian 3-manifold, from (2.13) we have

$$\mathscr{P}(\xi,X)Y = -\frac{1}{2}S(X,Y)\xi - \varepsilon g(X,Y)\xi. \quad (3.19)$$

Using (2.7) and (3.19), we obtain

$$\mathscr{Z}(\mathscr{P}(\xi,Y)U,V) = \left[-\frac{1}{2}S(Y,U) - \varepsilon g(Y,U)\right]\mathscr{Z}(V,\xi) \quad (3.20)$$

and

$$\mathscr{Z}(U,\mathscr{P}(\xi,Y)V) = \left[-\frac{1}{2}S(Y,V) - \varepsilon g(Y,V)\right]\mathscr{Z}(U,\xi). \quad (3.21)$$

The equations (3.18), (3.20) and (3.21) together give

$$\left[\frac{1}{2}S(Y,U) + \varepsilon g(Y,U)\right]\mathscr{Z}(V,\xi) + \left[\frac{1}{2}S(Y,V) + \varepsilon g(Y,V)\right]\mathscr{Z}(U,\xi) = 0. \quad (3.22)$$

Setting $V=\xi$ in (3.22) and then using (2.12) and (2.15), we have

$$(\psi - 2\varepsilon)[S(U,Y) + 2\varepsilon g(Y,U)] = 0. \quad (3.23)$$

This implies either $\psi = 2\varepsilon$ or $S(U,Y) = -2\varepsilon g(Y,U)$. If $\psi = 2\varepsilon$, then from (2.15) we obtain (3.7). It shows that M is an Einstein manifold. Therefore, in both of the cases, manifold M reduces to an Einstein manifold. Hence, we state the following:

Theorem 3.14. *Every projectively $\mathscr{Z}$-semisymmetric (ε)-para-Sasakian 3-manifold is an Einstein manifold.*

References

[1] T. Takahashi, Sasakain manifold with pseudo-Riemannian metric, *Tohoku Math. J.*, Second Series, 21(1969), 271–290.

[2] A. Bejancu and K. L. Duggal, Real hypersurfaces of indefinite Kaehler manifolds, *Int. J. Math. Math. Sci.,* 16(3)(1993), 545–556.

[3] K. L. Duggal, Space time manifolds and contact structures, *Int. J. Math. Math. Sci.,* 13(3)(1990), 545–554.

[4] K. L. Duggal and B. Sahin, Lightlike submanifolds of indefinite Sasakian manifolds, *Int. J. Math. Math. Sci.,* 2007, Art. ID 57585, 21 pages.

[5] R. Kumar, R. Rani and R. K. Nagaich, On sectional curvature of ε-Sasakian manifolds, *Int. J. Math. Math. Sci.,* 2007, Art. ID: 93562, 10 pages.

[6] X. Xufeng and C. Xiaoli, Two theorems on ε-Sasakian manifolds, *Int. J. Math. Math. Sci.,* 21(2)(1998), 249–254.

[7] U. C. De and A. Sarkar, On ε-Kenmotsu manifolds, *Hardonic J.*, 32(2)(2009), 231–242.

[8] M. M. Tripathi, E. Kihc, S. Y. Perktas, S. Keles, Indefinite almost para-contact metric manifolds, *Int. J. Math. Math. Sci.,* 2010 Art. ID 846195, 19 pages.

[9] S. Y. Perktas, E. Kilic, M. M. Tripathi and S. Keles, On (ε)-para-Sasakian 3-manifolds, *arXiv:0911.4786v2 [math.DG]* 1 Dec 2009.

[10] C. A. Mantica and L. G. Molonari, Weakly $\mathscr{Z}$ symmetric manifolds, *Acta Math. Hunger.,* 135(2012), 80–96.

[11] C. A. Mantica and Y. J. Suh, Pseudo Z symmetric Riemannian manifolds with harmonic curvature tensors, *Int. J. Geom. Meth. Mod. Phys.,* 9/1, 1250004(2012).

[12] C. A. Mantica and Y. J. Suh, Pseudo Z symmetric spacetimes, *J. Math. Phys.,* 55, 042502(2014).

[13] S. Mallick and U. C. De, $\mathscr{Z}$ tensor on N(k)-quasi-Einstein manifolds, *Kyungpook Math. J.,* 56(2016), 979–991.

[14] I. Sato, On a structure similar to the almost contact structure, *Tensor, N. S.,* 30(3)(1976), 219–224.

[15] I. Sato, On a structure similar to the almost contact structures II, *Tensor, N. S.,* 31(2)(1977), 199–205.

[16] S. Sasaki, On para-contact Riemannian manifolds, *TRU Math.,* 16(2)(1980), 75–86.

[17] K. Matsumoto, On Lorentzian Para-contact manifolds, *Bull. Yamagata Univ.,* 12(2)(1989), 151–156.

[18] Z. I. Szabo, Strucure theorems on Riemannian spaces satisfying $R(X,Y)\cdot R = 0$. The local version, *J. Differ. Geo.,* 17(1982), 531–582.

[19] R. Deszcz, On pseudosymmetric spaces, *Bull. Soc. Math. Belg., Ser. A* 44(1)(1992), 1–34.

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